Standard 1

Points, Lines, Angles and Planes

CORE STANDARD
Coordinate Geometry

Find slopes, lengths and midpoints of line segments using coordinate geometry. Use these measures to show whether shapes are similar or congruent and whether line segments are parallel or perpendicular. Find the equation of a circle in the coordinate plane.

[Standard Indicators: G.1.1, G.1.4, G.1.6, G.3.5]

CORE STANDARD
Angles and Lines

Understand the relationship between special angles created by parallel lines and transversals.

[Standard Indicator: G.1.3]

G.1.1 **Find the length of line segments in one- or two-dimensional coordinate systems, the slopes of line segments in two-dimensional coordinate systems, and the point that is a given fractional distance from one end of the segment to another.**

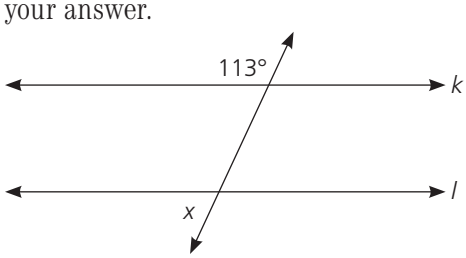
Example: Find the length of the line segment joining $A(3, 8)$ and $B(9, 0)$. Find the midpoint of this segment and the point that is one-third of the way from A to B.

G.1.2 **Construct congruent segments and angles, angle bisectors, perpendicular bisectors, and parallel and perpendicular lines by using appropriate geometric construction tools. Explain and justify the process used.**

Example: Construct the perpendicular bisector of a given line segment, justifying each step of the process.

G.1.3 **Recognize, use and justify the relationships between special pairs of angles formed by parallel lines and transversals.**

Example: In the diagram, the lines k and l are parallel. What is the measure of angle x? Explain your answer.



G.1.4 **Identify and apply properties of and theorems about parallel and perpendicular lines, write equations of parallel and perpendicular lines, and develop simple geometric proofs involving parallel and perpendicular lines.**

Example: Find an equation of a line perpendicular to $y = 4x - 2$ that contains the point (4, 1).

G.1.5 **Identify, justify and apply properties of planes.**

Example: Describe the intersection of plane R with parallel planes S and T.

G.1.6 **Represent geometric objects and figures algebraically using coordinates, use algebra to solve geometric problems, and develop simple coordinate proofs involving geometric objects in the coordinate plane.**

Example: Draw a triangle with vertices at (1, 3), (2, 5) and (6, 1). Draw another triangle with vertices (-3, -1), (-2, 1) and (2, -3). Are these triangles the same shape and size? Justify your answer.

G.1.7 **Describe the intersection of two or more geometric figures in the plane.**

Example: What is the maximum number of times two circles of the same size can intersect? Three circles? Six circles? Explain your reasoning.

Standard 2

Polygons

CORE STANDARD
Polygons

Find the sum of the measures of the interior and exterior angles of convex polygons. Deduce formulas relating lengths and sides, perimeters, and areas of regular polygons. Understand how limiting cases of such formulas leads to expressions for the circumference and the area of a circle.

[Standard Indicators: G.2.1, G.2.5]

CORE STANDARD
Congruence and Similarity

Develop simple geometric proofs involving congruent and similar polygons. Solve problems involving congruent and similar polygons and solids.

[Standard Indicators: G.2.3, G.2.7, G.2.12, G.4.2]

CORE STANDARD
Transformations

Predict and describe the results of translations, reflections and rotations. Describe a motion or series of motions that will show that two shapes are congruent.

[Standard Indicator: G.2.4]

CORE STANDARD
Geometric Proof and Reasoning

Understand the differences among supporting evidence, counterexamples and actual proofs. Be able to develop simple geometric proofs, providing reasons for each statement, involving the following topics:

- parallel lines and transversals;
- congruent and similar polygons, particularly triangles;
- circles; and
- geometric objects in the coordinate plane.

[Standard Indicators: G.1.4, G.2.7, G.2.14, G.3.6, G.5.3, G.5.4]

CORE STANDARD
Triangles

Prove the Pythagorean Theorem and its converse and use them to solve problems. Develop simple geometric proofs involving triangles. Define trigonometric functions in terms of angles of right triangles and use them to solve problems.

[Standard Indicators: G.2.14, G.2.16, G.2.17, G.2.19, G.2.21]

G

General

G.2.1 **Justifying the method used, find and use the sum of the measures of interior and exterior angles of convex polygons.**

Example: Calculate the measure of one interior angle of a regular octagon. Explain your method.

G.2.2 **Identify types of symmetry (i.e., line, point, rotational, self-congruences) of polygons.**

Example: $ABCD$ is a rhombus. Identify and describe its reflection and rotation symmetry.

G.2.3 Solve problems involving congruent and similar polygons.

Example: In the figures below, Figure A ≅ Figure B. Justify this statement: $\angle x \cong \angle y$.

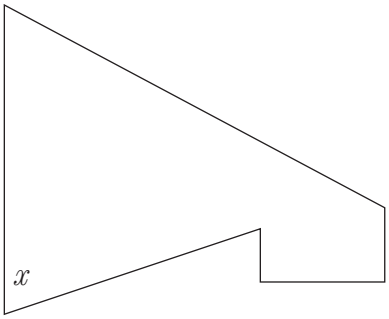


Figure A

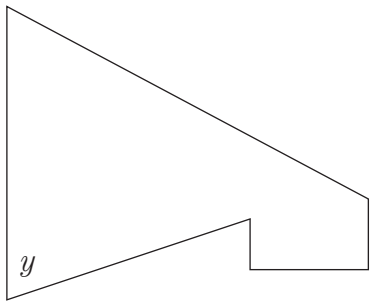


Figure B

G.2.4 Predict and describe the results of translations, reflections and rotations on polygons. Describe a motion or series of motions that will show that two shapes are congruent.

Example: Use a drawing program to create quadrilaterals and regular hexagons, octagons and pentagons. Under the drawings, describe which of the polygons tessellate. From your drawings, can you find a set of polygons in which all within the set tessellate? Show how you determined this.

G.2.5 Deduce formulas relating lengths and sides, perimeters, and areas of regular polygons. Understand how limiting cases of such formulas lead to expressions for the circumference and the area of a circle.

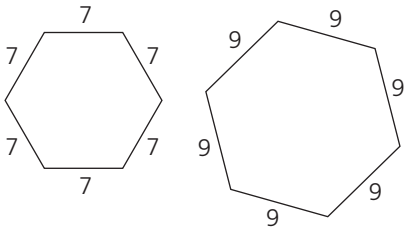
Example: Use trigonometric functions to find the perimeter and the area of a regular 12-gon that has been inscribed in a circle with radius r.

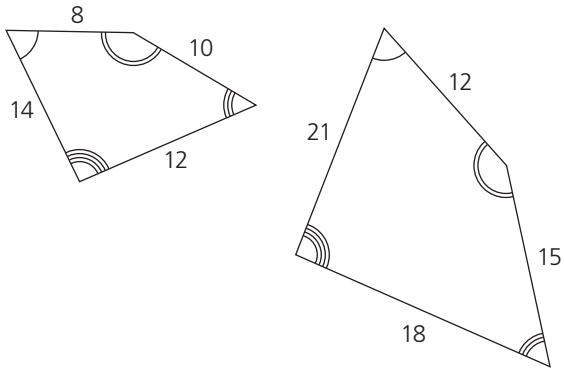
G.2.6 Recognize and use coordinate geometry to verify properties of polygons such as regularity, congruence and similarity.

Example: Is the polygon formed by connecting the points (2, 1), (6, 2), (5, 6) and (1, 5) a square?

G.2.7 Develop simple geometric proofs involving congruent and similar polygons and provide reasons for each statement.

Example: Prove that the following pairs of polygons are similar.





Quadrilaterals

G.2.8 Describe, classify and recognize relationships among the quadrilaterals, such as squares, rectangles, rhombuses, parallelograms, trapezoids and kites.

Example: Use a drawing program to create a square, rectangle, rhombus, parallelogram, trapezoid and kite. Judge which of the quadrilaterals has perpendicular diagonals and draw those diagonals in the figures. Give a convincing argument that your judgment is correct.

G.2.9 **Prove and apply theorems about parallelograms and trapezoids (including isosceles trapezoids) involving their angles, sides and diagonals. Prove that the given quadrilaterals are parallelograms, rhombuses, rectangles, squares or trapezoids (as appropriate).**

Example: Prove that both pairs of opposite sides of a parallelogram are congruent.

Triangles

G.2.10 **Define, identify, construct and solve problems involving perpendicular bisectors, angle bisectors, medians and altitudes in triangles.**

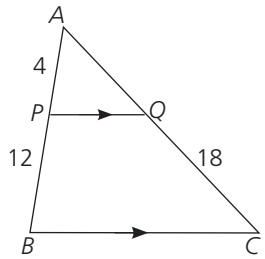
Example: Draw several triangles. Construct their angle bisectors. What do you notice?

G.2.11 **Construct triangles congruent to given triangles. Explain and justify the process used.**

Example: Construct a triangle given the lengths of two sides and the measure of the angle between the two sides.

G.2.12 **Use theorems to show if two triangles are congruent (i.e., SSS, SAS, ASA) or similar (i.e., AA, SAS, SSS).**

Example: In the example below, prove that ΔABC and ΔAPQ are similar and use the similar triangles to compute the length of $\overline{AQ}$.



G.2.13 **Prove and apply the triangle inequality theorem.**

Example: Can 7, 15 and 5 be the sides of a triangle? Explain how you know your answer is accurate.

G.2.14 **Develop simple geometric proofs involving triangles and provide reasons for each statement of the proof.**

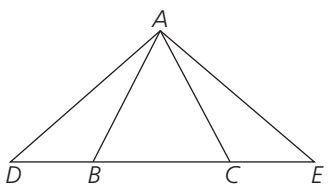
Example: Prove that:

- The sum of the angles in a triangle is 180°.
- The line joining the midpoint of two sides of a triangle is parallel to, and half the length of, the third side.
- The perpendicular bisectors of the sides of a triangle meet at a point that is the center of the circle and that contains the vertices of the triangle.

Isosceles Triangles

G.2.15 **Prove and apply the isosceles triangle theorem and its converse.**

Example: Given isosceles ΔABC and $DB \cong EC$, prove $\Delta DBA \cong \Delta ECA$.



Right Triangles

G.2.16 **Prove the Pythagorean Theorem and its converse and use them to solve problems, including problems involving the length of a segment in the coordinate plane.**

Example: Triangle DEF has vertices $D(2, 4)$, $E(0, 2)$, and $F(3, -1)$. Determine whether ΔDEF is a right triangle.

G.2.17 **Prove and apply the relationships that exist when an altitude is drawn to the hypotenuse of a right triangle.**

Example: In ΔABC with right angle at C, draw the altitude $\overline{CD}$ from C to $\overline{AB}$. Name all similar triangles in the diagram. Use these similar triangles to prove the Pythagorean Theorem.

G.2.18 **Use special right triangles (e.g., 30° - 60° and 45° - 45°) to solve problems.**

Example: An isosceles right triangle has one short side of 6 cm. Find the lengths of the other two sides.

G.2.19 **Define and use the trigonometric functions sine, cosine and tangent in terms of angles of right triangles.**

Example: In ΔABC, $\tan A = \frac{1}{5}$. Find $\sin A$ and $\cos A$.

G.2.20 **Deduce and apply the area formula $A = \frac{1}{2}ab \sin C$, where a and b are the lengths of two sides of a triangle and C is the measure of the included angle formed by the two sides.**

Example: Find the area of an equilateral triangle with sides five units long.

G.2.21 **Solve problems that can be modeled using right triangles, including problems that can be modeled using trigonometric functions. Interpret the solutions and determine whether the solutions are reasonable. Use technology as appropriate.**

Example: The force of gravity pulling an object down a hill is its weight multiplied by the sine of the angle of elevation of the hill. What is the force on a 3,000-pound car on a hill with a 1 in 5 grade? (A grade of 1 in 5 means that the hill rises one unit for every five horizontal units.)

Standard 3

Circles

G

CORE STANDARD
Circles

Define, deduce and use formulas for and prove theorems for:

- radius, diameter, arc, chord, secant and tangent;
- measures of arcs and related angles (central, inscribed, and intersections of secants and tangents); and
- circumference, arc length and areas of circles and sectors.

Determine how the graph of a circle changes if a, b and r are changed in the equation $(x - a)^2 + (y - b)^2 = r^2$.

[Standard Indicators: G.3.2, G.3.3, G.3.4, G.3.5, G.3.6]

CORE STANDARD
Coordinate Geometry

Find slopes, lengths and midpoints of line segments using coordinate geometry. Use these measures to show whether shapes are similar or congruent and whether line segments are parallel or perpendicular. Find the equation of a circle in the coordinate plane.

[Standard Indicators: G.1.1, G.1.4, G.1.6, G.3.5]

CORE STANDARD
Geometric Proof and Reasoning

Understand the differences among supporting evidence, counterexamples and actual proofs. Be able to develop simple geometric proofs, providing reasons for each statement, involving the following topics:

- parallel lines and transversals;
- congruent and similar polygons, particularly triangles;
- circles; and
- geometric objects in the coordinate plane.

[Standard Indicators: G.1.4, G.2.7, G.2.14, G.3.6, G.5.3, G.5.4]

G.3.1 **Construct the circle that passes through three given points not on a line. Construct tangents to circles. Circumscribe and inscribe circles. Justify the process used.**

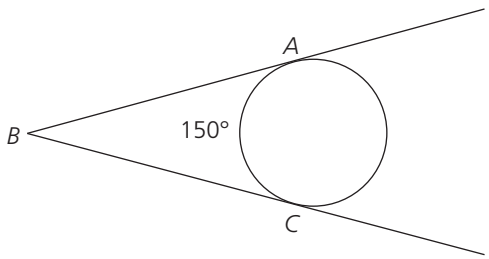
Example: Given a circle, find its center by drawing the perpendicular bisectors of two chords.

G.3.2 **Define, deduce and use formulas for, and prove theorems for radius, diameter, chord, secant and tangent.**

Example: What is the angle between a tangent to a circle and the radius at the point where the tangent meets the circle?

G.3.3 **Define, deduce and use formulas for, and prove theorems for measures of arcs and related angles (i.e., central, inscribed and intersections of secants and tangents).**

Example: Find the measure of $\angle ABC$ in the diagram below.



G.3.4 **Define, deduce and use formulas for, and prove theorems for measures of circumference, arc length, and areas of circles and sectors.**

Example: Use an appropriate theorem to find the sum of the interior angles of a convex n-gon.

G

G.3.5 Find the equation of a circle in the coordinate plane in terms of its center and radius and determine how the graph of a circle changes if a, b and r change in the equation $(x-a)^2+(y-b)^2=r^2$.

Example: Find the equation of the circle with radius 10 and center (6, -3).

G.3.6 Develop simple geometric proofs involving circles and provide reasons for each statement.

Example: Prove that an inscribed angle in a circle is half the measure of the central angle with the same arc.

Standard 4

Polyhedra and Other Solids

CORE STANDARD
Congruence and Similarity

Develop simple geometric proofs involving congruent and similar polygons. Provide reasons for each statement. Solve problems involving congruent and similar polygons and solids.

[Standard Indicators: G.2.3, G.2.7, G.2.12, G.4.2]

CORE STANDARD
Solids

Find and use measures of sides, volumes of solids and surface areas of solids. Relate these measures to each other using formulas.

[Standard Indicator: G.4.3]

G

G.4.1 Identify, justify and apply properties of prisms, regular pyramids, cylinders, right circular cones and spheres.

Example: Which of these properties of a cylinder is not true, and how do you know?

- The bases are congruent.
- The sections produced by the intersection of a cylinder and two parallel planes are congruent.
- The volume is the product of the area of the base and the altitude.
- The lateral area of a right circular cylinder is the product of the altitude and the base.

G.4.2 Solve problems involving congruent and similar solids.

Example: Explain how the surface area and volume of similar cylinders are related.

G.4.3 Find and use measures of sides, volumes and surface areas of prisms, regular pyramids, cylinders, right circular cones and spheres. Relate these measures to each other using formulas.

Example: A marble is dropped into a glass that is roughly a right cylinder with a 6 cm diameter. The water level rises 1 mm. What is the volume of the marble?

G.4.4 Visualize solids and surfaces in three-dimensional space when given two-dimensional representations, and create two-dimensional representations for the surfaces of three-dimensional objects.

Example: Make a net for a tetrahedron out of poster board and fold it to make the tetrahedron.

Standard 5

Geometric Reasoning and Proof

CORE STANDARD

Geometric Proof and Reasoning

Understand the differences among supporting evidence, counterexamples and actual proofs. Be able to develop simple geometric proofs, providing reasons for each statement, involving the following topics:

- parallel lines and transversals;
- congruent and similar polygons, particularly triangles;
- circles; and
- geometric objects in the coordinate plane.

[Standard Indicators: G.1.4, G.2.7, G.2.14, G.3.6, G.5.3, G.5.4]

G.5.1 Describe the structure of and relationships within an axiomatic system (e.g., undefined terms, definitions, axioms and postulates, methods of reasoning, and theorems).

Example: Do you prove axioms from theorems or theorems from axioms?

G.5.2 Recognize that there are geometries, other than Euclidean geometry, in which the parallel postulate is not true. Illustrate its counterparts in other geometries.

Example: Describe and illustrate at least one non-Euclidean geometry postulate.

G.5.3 Understand the differences among supporting evidence, counterexamples and actual proofs.

Example: Draw and label a figure for the conjecture, "If an angle bisector of a triangle is also an altitude, then the triangle is isosceles." Support your conjecture with supporting evidence. Then write a simple proof for your conjecture.

G.5.4 Develop simple geometric proofs (i.e., direct proofs, indirect proofs, proofs by contradiction and proofs involving coordinate geometry) using two-column, paragraphs and flow charts formats. Provide reasons for each statement in the proofs.

Example: Prove that the medians of a triangle meet at a point which is $\frac{2}{3}$ of the way from a vertex to the opposite side.

PROCESS STANDARDS

Indiana's Academic Standards for Mathematics describe the key content of each grade level and course, and students must develop conceptual understanding of this content. The American Diploma Project noted that, "beyond acquiring procedural mathematical skills with their clear methods and boundaries, students need to master the more subjective skills of reading, interpreting, representing and 'mathematicizing' a problem" (p. 55).

The National Council of Teachers of Mathematics has described five Process Standards that "highlight ways of acquiring and using content knowledge" (p. 29). The following Process Standards must be addressed throughout the learning and teaching of Indiana's Academic Standards for Mathematics in all grade levels in mathematics.

Problem Solving

- Build new mathematical knowledge through problem solving.
- Solve problems that arise in mathematics and in other contexts.
- Apply and adapt a variety of appropriate strategies to solve problems.
- Monitor and reflect on the process of mathematical problem solving.

Reasoning and Proof

- Recognize reasoning and proof as fundamental aspects of mathematics.
- Make and investigate mathematical conjectures.
- Develop and evaluate mathematical arguments and proofs.
- Select and use various types of reasoning and methods of proof.

Communication

- Organize and consolidate mathematical thinking through communication.
- Communicate mathematical thinking coherently and clearly to peers, teachers and others.
- Analyze and evaluate the mathematical thinking and strategies of others.
- Use the language of mathematics to express mathematical ideas precisely.

Connections

- Recognize and use connections among mathematical ideas.
- Understand how mathematical ideas interconnect and build on one another to produce a coherent whole.
- Recognize and apply mathematics in contexts outside of mathematics.

Representation

- Create and use representations to organize, record and communicate mathematical ideas.
- Select, apply and translate among mathematical representations to solve problems.
- Use representations to model and interpret physical, social and mathematical phenomena.

In addition, estimation, mental computation and technology are areas that need to be addressed at all grade levels in mathematics.

Estimation and Mental Computation

- Know and apply appropriate methods for estimating the results of computations.
- Use estimation to decide whether answers are reasonable.
- Decide when estimation is an appropriate strategy for solving a problem.
- Determine appropriate accuracy and precision of measurement in problem situations.
- Use properties of numbers and operations to perform mental computation.
- Recognize when the numbers involved in a computation allow for a mental computation strategy.

Technology

- Technology should be used as a tool in mathematics education to support and extend the mathematics curriculum.
- Technology can contribute to concept development, simulation, representation, communication and problem solving.
- The challenge is to ensure that technology supports, but is not a substitute for, the development of skills with basic operations, quantitative reasoning, and problem-solving skills.
  - Graphing calculators should be used to enhance middle school and high school students' understanding and skills.
  - The focus must be on learning mathematics and using technology as a tool rather than as an end unto itself.

References

American Diploma Project (2004). *Ready or Not: Creating a High School Diploma that Counts*. Washington, D.C.: Achieve, Inc.

National Council of Teachers of Mathematics (2000). *Principles and Standards for School Mathematics*. Reston VA: author.

NOTES